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Exploring Accelerated Cosmic Expansion: Dark Energy Models and Modified Gravity Theories in f(R, T) Cosmology

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Abstract

This paper explores the recent advancements in understanding the accelerated expansion of the universe, emphasizing the role of dark energy (DE) and alternative gravitational theories. Observations, such as those from the Supernova Cosmology Project, the Wilkinson Microwave Anisotropy Probe (WMAP), and the Sloan Digital Sky Survey (SDSS), have consistently indicated an ongoing accelerated expansion. The universe is currently composed of approximately 68.3% dark energy, 26.8% dark matter, and 4.9% baryonic matter, with DE being a primary driver of cosmic acceleration. We discuss the role of the cosmological constant, quintessence, phantom, k-essence, tachyons, and Chaplygin gas as potential representations of DE. The paper further explores modified theories of gravity, including f(R), f(G), f(T), and f(R, T) gravity, as alternative explanations for cosmic acceleration. Specifically, we examine the f(R, T) gravity model, utilizing the field equations and a proposed variation in the Hubble parameter H as a function of cosmic time. The approach presented provides insights into the behavior of the universe's expansion based on empirical data and generalized gravitational actions. This study contributes to our understanding of cosmic acceleration, exploring both dark energy models and modifications to Einstein's general relativity. Future research directions involve refining these models to better align with observational data and improve the understanding of the fundamental nature of dark energy.

Keywords: accelerated expansion, dark energy, cosmological constant, modified gravity, f(R, T) gravity, Hubble parameter, cosmic evolution, Einstein gravity, observational cosmology.

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1. Introduction

1.1 LRS Bianchi type I cosmological model with late time acceleration

Modern cosmological researches have provided ample data that points to the fact that the acceleration of our universe and has now reached a new vision to establish revolutionary advancements on account of the current accelerated expansion. Findings by the likes of the supernovae cosmology project have provided revolutionary evidence in the favor of the theory of cosmic acceleration of the universe [1,3] and several studies like those of the distant supernovae [4,5], large scale structure (LSS), fluctuation of the cosmic microwave background radiation [6-7]. The Wilkinson microwave anisotropy probe [8], the Sloan Digital Sky Survey [SDSS] [9] and the Chandra x-ray observatory indicated that the universe is undergoing an accelerated expansion. These observations indicate a change corresponding to time from an early deceleration of the cosmic expansion of the universe.

One way of explaining this phenomenon of the expanding universe is that it is dominated by a negative gravity like matter labelled dark energy (DE), which seems to be responsible for the accelerating rate of expansion of the universe [10]. Recent cosmological studies indicate that the universe is embodied with 68.3%DE, 26.8% dark matter and 4.9% baryonic matter. The cosmological constant is a worthy candidate for DE although other representations like as quintessence [11], phantom [12], k-essence [13], tachyons [14], and chaplygin gas [15] also exist. The second approach is to generalization of the Einstein gravity model of general relativity, and considers more universal actions to describe the gravitational field. Let's take an example of the standard Einstein Hilbert action that has been replaced by an arbitrary function of the Ricci scalar in one approach. A fundamental theoretical hurdle for gravitational theories is giving an explanation for the late time accelerations of the universe and the effective cause related to DE. Based on changes to the action, many alternative theories of gravity have come into existence such as $f(R)$,$f(G)$,$f(T)$ and $f(R,T)$ gravity. The cosmological evolution for DE models was discussed by Sharif and Azeem [16] in $f(T)$ gravity. Both cosmic inflation and an explanation of DE including the present cosmic acceleration [17] is produced by $f(R)$ modified theory. The anisotropic

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cosmological models in $f(R,T)$ gravity with variable deceleration parameters has been studied upon by Sahoo et al [18] recently. In this paper, we dealt with the basic formalism of the $f(R,T)$ gravity field equations along with some essential reviews. The field equations in $f(R,T)$ gravity are solved by assuming that $f(R,T) = R + 2f(T)$ and the relation between Hubble parameter H and cosmic time t as[19].

$$H = m + n\coth t, \tag{1.1}$$

where m and n are positive constants. This arrangement of Hubble's parameter H as a function of cosmic time t, we propose is makeshift arrangement since it is built upon the desired behaviour of the universe, rather than from a known field theory. Berman [20] also studied cosmological models that proposed a variation law for the Hubble parameter without specifying the physics behind such a choice which yields a constant value of deceleration parameter.

1.2 Basic formulation of $f(R,T)$ gravity

The action of $f(R,T)$ gravity is given by Harko et al.[19]

$$S = \int \left(\frac{f(R,T)}{16\pi} + S_m \right) \sqrt{-g} dx^4 . \tag{1.2}$$

Here $f(R,T)$ is an arbitrary function of Ricci tensor R and trace energy-momentum tensor T. Harko et al.[19] explored the following conditions.

$$R + 2f(T),$$

$$f(R,T) = f_1(R) + f_2(T)$$

$$f1(R) + f2(R)f3(T).$$

In particular paper, we assume the form of $f(R,T) = R + 2f(T)$, and take $f(T) = \lambda T$, where λ is coupling constant of $f(R,T)$gravity. We can rewrite equation (2.2) using this condition as

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$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R,T)$$

$$= 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}, \quad (1.3)$$

where is D'Alembertian operator and defined as ($= \nabla^i\nabla_j$), and also Θ_{ij} is defined as

$$\Theta_{ij} = g^{lm}\frac{\delta T_{lm}}{\delta g^{ij}}. \quad (1.4)$$

We consider the matter content in the universe to be perfect fluid, Θ_{ij} becomes

$$\Theta_{ij} = -2T_{ij} - pg_{ij}, \quad (1.5)$$

here T_{ij} is the energy momentum tensor with perfect fluid defined as

$$T_{ij} = (\rho + p)u_iu_j - pg_{ij}, \quad (1.6)$$

where ρ and p are ED and cosmic pressure. u^i is four velocity vector and satisfying the condition $u^iu_i = 1$.

The field equation attains the form

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f_TT_{ij} + [f(T) + 2pf_T]g_{ij}. \quad (1.7)$$

or

$$R_{ij} - \frac{1}{2}Rg_{ij} = (8\pi + 2\lambda)T_{ij} + [T + 2p]\lambda g_{ij}, \quad (1.8)$$

where $G = c = 1$.

2. Line element and field equations

The spatially homogeneous and anisotropic LRS Bianchi type-I universe is described by the line element

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 + dz^2), \quad (2.1)$$

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where A,B are functions of cosmic time t. With the help of equations (2.1) and (2.2) in the field equations , The average scale factor (a) for the Bianchi type-I space time is defined as

$$a^3 = AB^2. \tag{2.2}$$

The Hubble parameter (H), shear scalar (σ), anisotropy parameter (A_m) and deceleration parameter (q), defined as

$$H = \frac{\dot{a}}{a}, \tag{2.3}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij},$$

$$A_m = \frac{2\sigma^2}{3H^2}, \tag{2.4}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$$

.

On integrating we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k}{a^3}, \tag{2.5}$$

where k is a constant of integration.

$$\frac{\dot{A}}{A} = \frac{\dot{a}}{a} + \frac{2k}{3a^3} ,$$

$$\frac{\dot{B}}{B} = \frac{\dot{a}}{a} - \frac{k}{3a^3}. \tag{2.6}$$

3. Solution of the field equations

For the hypothesis, the physical parameters, scalar factor (a), expansion scalar (θ), shear scalar (σ) and deceleration parameter (q) are given by

$$a = e^{mt}(\sinh t)^n, \tag{3.1}$$

$$\theta = 3(m + n\coth t),$$

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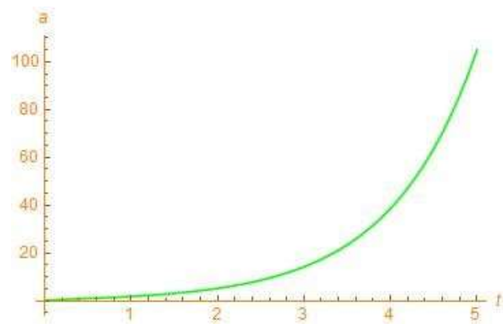
$$\sigma = \frac{k}{e^{3mt}(\sinh t)^{3n}} \tag{3.2}$$

$$q = -1 + \frac{n}{(m\sinh t + n\cosh t)^2}$$



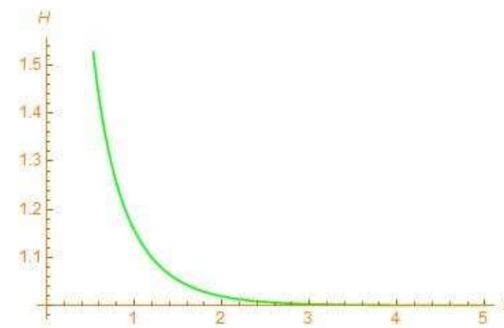


Fig.1- Scalar factor vs time. **Fig.2- Hubble parameter vs time**

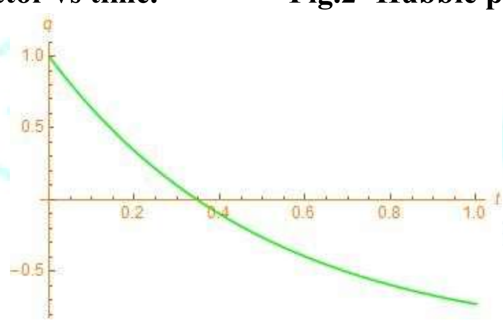


Fig.3 Deceleration parameter vs time.

We perceive that as $t = 0$ the scale factor is zero and expansion scalar θ and shear scalar σ are infinity and $t \to \infty$ the scalar factor and shear scalar are infinity and expansion scalar is zero. The proposed model is in line with the Big-Bang theory. The deceleration parameter $q = -1 + \frac{1}{n} > 0$,where $0 < n < 1$ at $t = 0$ and for $t = \infty, q = -1$. Thus, the model represents initial deceleration phase and late time acceleration phase of expansion.

As $t \to \infty$, the anisotropy $\frac{\sigma}{\theta} \to 0$. Therefore the model approaches isotropy at late time.
The pressure and energy density are given by

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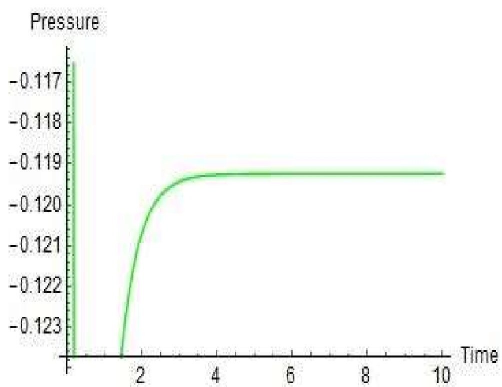
$$p = \frac{-1}{[(8\pi+3\lambda)^2-\lambda^2]}\left\{-2(8\pi+3\lambda)ncosech^2 t + (24\pi+6\lambda)(m+n\,t)^2 + \frac{(8\pi+4\lambda)k^2}{3\sinh^{6n} te^{6mt}}\right\},$$

$$\rho = \frac{1}{[(8\pi+3\lambda)^2-\lambda^2]}\left\{-2\lambda ncosech^2 t + (24\pi+6\lambda)(m+n\,t)^2 - \frac{(8\pi+4\lambda)k^2}{3\sinh^{6n} te^{6mt}}\right\},$$



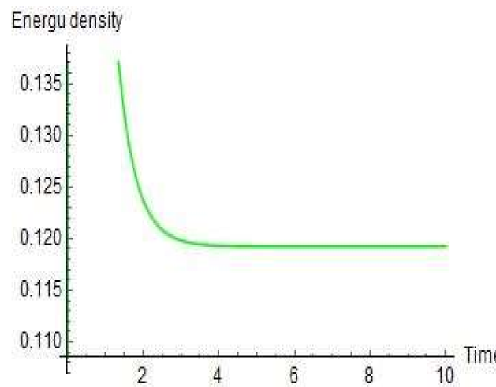


Fig.4- Pressure vs time. **Fig.5- Energy density vs time**

We observe that the energy density is a positive and decreasing function of cosmic t *t,* and pressure is negative, the negative pressure is responsible for accelerating universe in $f(R,T)$ theory.

4. Discussion

The presented LRS Bianchi type-I cosmological model in the framework of f(R,T) gravity explores the phenomenon of late-time acceleration of the universe. The model effectively demonstrates both early deceleration and late-time acceleration phases, consistent with observational findings. This suggests that it can capture the essential features of the universe's dynamical behavior. The value of the deceleration parameter q initially being positive (indicating deceleration) and eventually tending towards -1 (suggesting acceleration) highlights that the universe underwent a transition from an early decelerated state to a late-time accelerated state. This observation aligns with recent cosmological data, which indicate an accelerated expansion driven by dark energy.

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The findings of this study show that the physical parameters such as scale factor a, expansion scalar θ and shear scalar σ evolve in a manner that conforms to the big bang theory. Specifically, as t=0, both θ and σ tend to infinity, which signifies the initial singularity. At late times (t→∞t), the scale factor grows unboundedly while both θ and σ approach zero, indicating a gradual decrease in expansion rate and anisotropy.

The negative pressure in the model, which is responsible for the acceleration of the universe, is a notable outcome. This feature is directly linked to the dark energy component in the f(R,T) theory. Furthermore, the decrease in energy density as cosmic time increases is consistent with the physical picture of a universe transitioning towards a more homogeneous and isotropic state. The anisotropy parameter approaching zero as t→∞t \rightarrow \inftyt→∞ signifies that the universe eventually attains an isotropic configuration, which is a key aspect of its large-scale structure.

Figures presented in the model, including the scale factor, Hubble parameter, deceleration parameter, pressure, and energy density as functions of cosmic time, visually illustrate these behaviors, offering a comprehensive understanding of the universe's dynamical evolution under the proposed framework. The proposed f(R,T) gravity model seems to effectively capture the dynamics of dark energy, allowing the universe to transition from early deceleration to late-time acceleration.

5. Conclusion

In this paper, an LRS Bianchi type-I cosmological model is analyzed in the context of f(R,T) gravity with a specific choice of the functional form f(R,T)=R+2λTf(R,T) = R + 2\lambda Tf(R,T)=R+2λT. The model effectively addresses the observed phenomenon of the universe's late-time acceleration and provides insights into its evolutionary dynamics. The Hubble parameter is chosen to reflect the desired cosmological behavior, transitioning from an initial deceleration phase to a late-time acceleration phase, which is consistent with recent observational data.

The key conclusions are:

1. **Cosmic Expansion Dynamics**: The proposed model shows that the universe transitions from an initial deceleration to a late-time acceleration. The deceleration parameter qqq changes from

a positive value in the early stages to -1 as time progresses, which is consistent with the observed accelerated expansion driven by dark energy.

2. **Initial Singularity and Big Bang Consistency**: At t=0t = 0t=0, the scale factor aaa is zero, while the expansion scalar θ\thetaθ and shear scalar σ\sigmaσ diverge to infinity, suggesting the initial singularity in line with the big bang theory.
3. **Evolution of Physical Parameters**: The analysis shows that as time increases, the scale factor grows, the expansion scalar and shear scalar diminish, and the anisotropy parameter approaches zero. This indicates that the universe evolves towards a homogeneous and isotropic state.
4. **Negative Pressure as the Cause of Acceleration**: The pressure in the model is negative, which plays a crucial role in driving the accelerated expansion of the universe. This feature is attributed to the influence of dark energy, consistent with observations indicating a universe predominantly made of dark energy.

Overall, the study contributes to the understanding of the late-time acceleration of the universe through an LRS Bianchi type-I model in the f(R,T) gravity framework. The model aligns well with current cosmological observations and offers an explanation for the transition between different phases of the universe's expansion, driven by dark energy and influenced by negative cosmic pressure.

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